RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

P.G. SECOND YEAR
M.A./M.SC. THIRD SEMESTER (July – December), 2011
Mid-Semester Examination, September, 2011

Date : 19/09/2011 **MATHEMATICS**
Time : 11 am – 3 pm Paper : I Full Marks : 90

[Use separate answer scripts for each group]

Group – A

1. Answer **any five** of the following : [5×3 = 15]

a) Let f(z) be an entire function. If Re f(z) = u(x, y) is bounded for all (x, y) in the complex plane, show that u(x, y) & v(x, y) are constant functions.

b) Using Residue theorem, evaluate $\int_C \frac{dz}{(z-3)(z^5-1)}$ where C : |z| = 2 in the positive sense of orientation.

c) Determine the domain of Convergence of the series $\sum_{n=1}^{\infty}\frac{e^{in}}{(z+1)^n}+\sum_{n=0}^{\infty}\frac{(z+1)^n}{e^{in+\frac{1}{2}}}$.

d) Use Contour integration technique to find the value of $\int_0^{2\pi}\frac{d\theta}{2+\cos\theta}$.

e) Evaluate $\int_C \frac{ze^z dz}{(4z+\pi i)^2}$ where C is |z| = 1.

f) For the function f(z) = $\sin\left(\frac{1}{1-z}\right)$, show that the accumulation point of the set of its zeros is an essential singularity of f(z).

g) Suppose the co-efficients of the power series $\sum_{n=0}^{\infty}a_n z^n$ are given by the recurrence relation $3a_n+4a_{n-1}-a_{n-2}=0$ for $n\geq 2$, where a_0 = 1, a_1 = –1. Find the radius of the convergence of the series.

h) If f is analytic in a neighbourhood of infinity & f is bounded there, show that the Laurent Co efficients a_n = 0 for $n\in\mathbb{N}$.

i) Using appropriate form of Argument principle, evaluate $I=\frac{1}{2\pi i}\int_C\frac{f'(z)}{f(z)}dz$ where $C=\{z:|z-1-i|=2\}$ and $f(z)=\frac{z-2}{z(z-1)}$.

j) Find the number of roots of the equation z^6 –6z + 10 = 0 in the circle |z| < 1.

k) Expand f(z) = –1 / {(z–1)(z–2)} as a power series in z in $1<|z|<2$.

2. Answer **any two** of the following : [2×5 = 10]

a) Show that a necessary and sufficient condition for f(z) to have a pole at z_0 is that $\lim_{Z\to z_0}|f(z)|=\infty$.

b) For any closed curve γ and $a\notin\gamma$, show that $\frac{1}{2\pi i}\int_\gamma\frac{dz}{z-a}$ is an integer.

c) Let z_0 be an isolated essential singularity of f(z). Show that given any complex number c and $\varepsilon > 0$, there exists $\delta > 0$ such that $|f(z) - c| < \varepsilon$ for all $Z \in N'(z_0, \delta)$.

d) If f has a zero of order m in at z = a and a pole of order n at z = b, show that $\text{Res}\left\{\frac{f'(z)}{f(z)}, a\right\} = m$ and $\text{Res}\left\{\frac{f'(z)}{f(z)}, b\right\} = -n$.

Group – B

3. Answer **any six** of the following : [6×5 = 30]

a) Define degree of a vertex in a graph. Define minimum degree $\delta(G)$ and maximum degree $\Delta(G)$ of a graph G. Show that in a graph G the average vertex degree is $\frac{2e(G)}{n(G)}$ and hence show that $\delta(G) \le \frac{2e(G)}{n(G)} \le \Delta(G)$. [1+1+1+2]

b) Define Euler graph. Prove that a connected graph having all the vertices of even degrees is an Euler graph. [1+4]

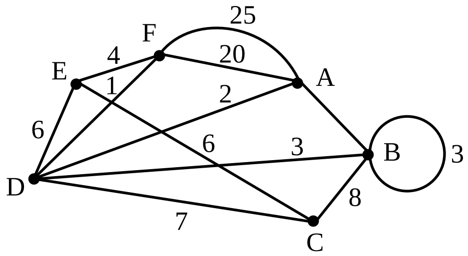
c) Prove that a graph G with p-vertices and satisfying the condition $\delta(G) \ge \frac{p-1}{2}$ is connected, where $\delta(G)$ is the minimum degree of the graph G. [5]

d) If a graph G has a Hamiltonian cycle, then show that for each non empty set $S \subseteq V(G)$, the graph G\S has at most |S| components. [5]

e) Define tree. Prove that a tree with n vertices has (n – 1) edges. [1+4]

f) Define spanning tree in a graph. Prove that every connected graph G contains a spanning tree. [1+4]

g) By Prim's Algorithm find a minimal spanning tree of the following graph and find the corresponding minimum weight of the spanning tree. [5]



h) For any connected plane graph with n-vertices, e-edges and f-faces show that n – e + f = 2. [5]

i) Prove that the graph $K_{3,3}$ is non planar. [5]

Group – C

4. Answer **any two** of the following : [5×2 = 10]

a) Show that $y^2u_{xx} + 2xyu_{xy} + x^2u_{yy} - xu_x - yu_y = 0$ is parabolic type and reduce its canonical form.

b) Define Cauchy problem for second order semi-linear PDE and prove that the characteristic curves of the above equation are invariant with respect to any transformation.

c) Find the D'Alembert's solution of one dimensional wave equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ given u(x, 0) = f(x) and $\frac{\partial u}{\partial t}(x, 0) = g(x)$. Also prove that the solution is stable.

5. Answer **any two** of the following : [5×2 = 10]

a) Construct the cubic spline interpolant to $f(x) = x^4 + 2x$ with nodes at –1, 0, 1 subject to the clamped boundary conditions. [5]

b) Using Romberg integration calculate $\int_0^{1/2} \frac{x}{\sin x}\,dx$ with step length $h = 0{\cdot}25$. [5]

c) Explain cubic spline Interpolation. [5]

Group – D

6. Answer **any three** of the following : [3×5 = 15]

a) If a particle falls freely from rest from a height h above the earth's surface, show that the particle suffers an easterly deviation of amount $\frac{1}{3} g\,\omega \left(\frac{2h}{g}\right)^{3/2} \cos\lambda$ where λ is the latitude of the place and ω is the angular velocity of the earth about its axis. [5]

b) If the Lagrangian L does not contain time t explicitly and if the force acting on the system be conservative, show by using Lagrange's equations of motion that total energy is conserved. [5]

c) Show that Lagrange's equations in the form $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_r}\right) - \frac{\partial T}{\partial q_r} = Q_r$ can also be written as $\frac{\partial \dot{T}}{\partial \dot{q}_r} - 2\frac{\partial T}{\partial q_r} = Q_r$ where T is the kinetic energy and Q_r is generalized force component associated with q_r.[5]

d) A particle of mass 4m is attached to a string which passes over a smooth fixed pulley. The other end of the string is fastened to a smooth pulley of mass m, over which passes a second string attached to masses m and 2m. The system starts from rest. Apply Lagrange's equations of motion to find the downward acceleration of the particle of mass 2m. [5]

e) What are ignorable coordinates? Use Routh's procedure to solve the problem of planetary motion by the method of ignoration of coordinates. [5]